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NORMAL FORMS

The Problem of finding whether a given statement is tautology or contradiction or satisfiable in a finite number of steps is called the Decision problem.

For Decision Problem, Construction of truth table may not be practical.

We Consider an alternative procedure known as the reduction to normal forms. $(p \leftrightarrow q) \wedge q$
(The Standard forms are called Normal forms).

There are two such forms:-

NORMAL FORMS

Disjunctive Normal forms	Conjunctive Normal forms
(DNF) (Sum) $\vee$	(Product) $\wedge$
(Sum of elementary products)	(Product of Elementary Sum)
Eg:- $(P \wedge Q) \vee (\neg P \wedge Q)$	Eg: $(P \vee Q) \wedge (\neg P \vee \neg Q)$

Elementary Product

- Product of the variables and their negations.

Example :- p and q are two atomic variables
$\neg p \wedge q$, $p \wedge \neg p$, $q \wedge \neg p$.

Elementary Sum

Sum of the variables and their Negations.

Example:-
$\neg p \vee q$, $p \vee \neg p$, $q \vee \neg p$, $\neg q \vee p \vee \neg p$.

(i) Disjunctive Normal Forms :-

A formula which is equivalent to a given formula and which consists of a sum of elementary products is called a disjunctive normal form.

Q: Obtain disjunctive Normal form :-

$p \wedge (p \to q)$

$\Rightarrow p \wedge (\neg p \vee q)$

$\Rightarrow (p \wedge \neg p) \vee (p \wedge q)$ (using distributive law).

(It is DNF)

Rules
$p \to q = \neg p \vee q$
$p \leftrightarrow q = (p \wedge q) \vee (\neg p \wedge \neg q)$ (DNF)

(ii) Conjunctive Normal forms :-

A formula which is equivalent to a given formula and which consists of a product of elementary sum is called a CNF.

Q: Obtain CNF of :-

$(p \to q) \wedge (q \vee (p \wedge r))$

$= (\neg p \vee q) \wedge (q \vee (p \wedge r))$

$= (\neg p \vee q) \wedge (q \vee p) \wedge (q \vee r)$

It is CNF

Rules
$p \to q = \neg p \vee q$